Eigenvalues for a Centrally Loaded Circular Cylindrical Cavity*

D. M. BOLLE†, MEMBER, IRE

Summary—**The general availability of large high-speed computers has made the use of series expansions in complicated field theory problems feasible even when these series do not converge rapidly. This paper outlines the method and illustrates its application to the solution of a problem in cylindrical coordinates. At the same time, the errors inherent in this method of solution are indicated and estimates are made of their magnitudes. A comparison of the method with Slater's perturbation theory is made.**

Introduction

CONSIDERABLE WORK has already been performed [1]–[3] on cylindrical waveguides using the method first proposed by Hahn [4]. This method consisted of splitting a bounded region into a number of subspaces, solving the wave equation within each subspace, and matching the resulting solutions across the interfaces of the subregions. In prior work, the parameters of prime interest were the shunt impedances introduced at the discontinuities in waveguides. This paper will discuss a method for determining the eigenvalues for a centrally loaded circular cylindrical cavity. The eigenvalues, being characteristics of the cavity system and fully describing it, are the parameters [7] most sensitive to dimensional variation. At the same time, the accuracy of any proposed method for obtaining them may be readily and accurately evaluated through comparison with experimental results.

This method of calculation also allows the rapid evaluation of a proportionality constant k_s which may be introduced into Slater's perturbation theory as a first order correction. If Slater's perturbation theory is written in the form [7]

$$\frac{\omega^2 - \omega_0^2}{\omega_0^2} = \frac{\int_{\Delta t} (\mu \overline{H} \cdot \overline{H} - \epsilon \overline{E} \cdot \overline{E}) dv}{\int_{V_t} (\mu \overline{H} \cdot \overline{H} + \epsilon \overline{E} \cdot \overline{E}) dv} \cdot k_s = K_i k_s,$$

where

k_s = a constant depending on the geometry of the perturbing object and the mode under consideration,[1]
Δt = volume of the perturbing object,
V_t = total cavity volume,
ω_0 = resonant frequency of the unperturbed cavity,

then

$$\frac{\omega}{\omega_0} \cong \frac{k_s K_i}{2} + 1$$

provided Δt is sufficiently small. Therefore, the evaluatioh of the slope of a plot of the variation of resonant frequencies with changes in the volume of the disturbing object from the method to be discussed in this paper will yield the constant k_s.

Theory

Fig. 1 shows the cross section of the cavity to be considered. The cavity is presumed constructed from a perfect conductor and filled with an ideal dielectric. The eigenvalues to be determined will be those of the TM_{0ij}

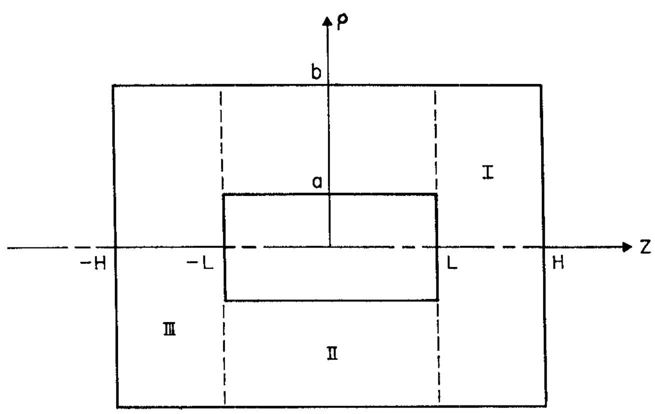

Fig. 1.

* Received by the PGMTT, July 18, 1961; revised manuscript received, November 14, 1961.
† Department of Elec. Engrg., Purdue University, Lafayette, Ind.

[1] See for example, L. C. Maier and J. C. Slater, "Field measurements in resonant cavities," *J. Appl. Phys.*, vol. 23, pp. 68 and 78; January, 1952.

modes (j even). The region within the cavity is divided into subregions I, II and III. Through symmetry, we may restrict ourselves to the region $0 \leq z \leq H$ and, consequently, only the subregions I and II are of interest. The solutions of the wave equation

$$(\nabla^2 + k_0^2)E_z = 0 \tag{1}$$

in the subregions I and II, when E_z is assumed symmetric in region II about the plane $z=0$ and with respect to the ϕ coordinate, are

$$E_{zI} = \sum_{m=1}^{\infty} a_m J_0(k_m \rho)[\cos \beta_m z + C_m \sin \rho_m z] \tag{2}$$

and

$$E_{zII} = \sum_{n=1}^{\infty} b_n Z_0(k_n \rho) \cos \Gamma_n z, \tag{3}$$

where

$$Z_0(k_n \rho) = J_0(k_n \rho) + R_n N_0(k_n \rho) \tag{4}$$

and k_m, β_m, k_n, and Γ_n are constants of separation such that

$$k_0^2 = k_m^2 + \beta_m^2 = k_n^2 + \Gamma_n^2. \tag{5}$$

These field modes are the ones of interest in linear particle accelerators. Since $E_{\rho II} = 0$, for $z=0$, the central structure may be supported by slender columns in the plane $z=0$ without disrupting the field significantly.

The procedure used in the analysis is as follows. The general solutions to the wave equation for the subspaces are written in the coordinate system applicable to the particular geometry under consideration. The number of subspaces then determines the number of infinite sets of constants in the series expansions for the field components as well as the number of unknown constants of separation that must be evaluated through an application of the boundary conditions. After applying the boundary conditions and matching the field components across the interfaces between the subspaces, there will remain a number of infinite sets of linear simultaneous equations. After eliminating the unknown constants, these yield a single equation that may then be solved for the remaining unknown constant of separation and give the eigenvalues of the system.

Applying the boundary conditions

$$E_{\rho I} = 0, \quad z = H, \quad 0 \leq \rho \leq a; \tag{6}$$

$$E_{zI} = 0, \quad \rho = b, \quad L \leq z \leq H; \tag{7}$$

$$E_{zII} = 0, \quad \rho = a, \quad 0 \leq z \leq L; \tag{8}$$

$$E_{zII} = 0, \quad \rho = b, \quad 0 \leq z \leq L; \tag{9}$$

to (2) and (3) gives

$$E_{zI} = \sum_{m=1}^{\infty} a_m J_0(p_{0m}\rho/b) \cos \beta_m(H-z)/\cos \beta_m H, \tag{10}$$

$$E_{zII} = \sum_{n=1}^{\infty} b_n Z_0(v_{0n}\rho/b) \cos \Gamma_n z, \tag{11}$$

where the p_{0m} are the zeros of

$$J_0(p) = 0$$

and the v_{0n} are the zeros of

$$J_0(v)N_0(v/r) - J_0(v/r)N_0(v) = 0,$$

where

$$r = b/a, \qquad r > 1. \tag{12}$$

The last step is to apply the boundary conditions across the interface $z=L$, $a \leq \rho \leq b$, *i.e.*, to match the field components across the interface between regions I and II. At the interface we equate the radial and longitudinal components of the field and use the fact that, over the intervals considered, the Bessel functions and linear combination of Bessel functions are orthogonal. This allows the relationships between the expansions coefficients to be obtained. For the continuity of the longitudinal component at $z=L$ and $a \leq \rho \leq b$ we may write

$$\int_a^b E_{zI}\rho Z_0(v_{0p}\rho/b)d\rho = \int_a^b E_{zII}\rho Z_0(v_{0p}\rho/b)d\rho. \tag{13}$$

For the radial electric field component we may write

$$\int_0^b E_{\rho I}\rho J_0(p_{0q}\rho/b)d\rho = \int_0^a E_{\rho I}\rho J_0(p_{0q}\rho/b)d\rho + \int_a^b E_{\rho I}\rho J_0(p_{0q}\rho/b)d\rho,$$

but

$$E_{\rho I} = 0 \quad \text{at} \quad z = L, \quad 0 \leq \rho \leq a,$$

and

$$E_\rho = E_{\rho II} \quad \text{at} \quad z = L, \quad a \leq \rho \leq b.$$

Therefore

$$\int_0^b E_{\rho I}\rho J_0(p_{0q}\rho/b)d\rho = \int_a^b E_{\rho II}\rho J_0(p_{0q}\rho/b)d\rho. \tag{14}$$

From (13) and (14) we obtain the following results:

$$B_p = \frac{2rv_{0p}Z_1(v_{0p}/r)}{[r^2 Z_1^2(v_{0p}) - Z_1^2(v_{0p}/r)]} \cdot \sum_{m=1}^{\infty} A_m \frac{J_0(p_{0m}/r)}{(p_{0m}^2 - v_{0p}^2)} \tag{15}$$

$$R_{Aq}A_q = \frac{2p_{0q}J_0(p_{0q}/r)}{rJ_1^2(p_{0q})} \cdot \sum_{n=1}^{\infty} B_n R_{Bn} \frac{Z_1(v_{0n}/r)}{(p_{0q}^2 - v_{0n}^2)}, \tag{16}$$

where

$$A_m = a_m \frac{\cos \beta_m(H-L)}{\cos \beta_m H}, \quad R_{Am} = \frac{b\beta_m}{p_{0m}} \tan \beta_m(H-L) \tag{17}$$

$$B_n = b_n \cos \Gamma_n L, \qquad R_{Bn} = \frac{b\Gamma_n}{v_{0n}} \tan \Gamma_n L. \tag{18}$$

But the $B_p = B_n$, thus eliminating the B_n from (16) using (15), gives

$$\frac{1}{4}R_{Aq}A_qJ_1^2(p_{0q}) = \sum_{n=1}^{\infty} R_{Bn}\frac{v_{0n}Z_1^2(v_{0n}/r)}{[r^2Z_1^2(v_{0n}) - Z_1^2(v_{0n}/r)]} \cdot \sum_{m=1}^{\infty} A_m \frac{p_{0q}J_0(p_{0m}/r)J_0(p_{0q}/r)}{(v_{0n}^2 - p_{0m}^2)(v_{0n}^2 - p_{0q}^2)} \cdot \quad (19)$$

The eigenvalues, k_{0n}, of the cavity will therefore be given by the roots of the secular determinant of (19);

$$\begin{vmatrix} (h_{11} - x_1) & h_{12} & h_{13} & \cdots \\ h_{21} & (h_{22} - x_2) & h_{23} & \cdots \\ h_{31} & h_{32} & (h_{33} - x_3) & \cdots \\ h_{41} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \end{vmatrix}, \quad (20)$$

where

$$h_{mq} = \sum_{n=1}^{\infty} \Gamma_n \tan(\Gamma_n L)\cdot z_n \cdot h_{nmq} = h_{qm}, \quad (21)$$

$$x_q = \beta_q \tan \beta_q(H - L)\cdot J_1^2(p_{0q})/4p_{0q}, \quad (22)$$

and

$$h_{nmq} = \frac{J_0(p_{0m}/r)J_0(p_{0q}/r)}{(v_{0n}^2 - p_{0m}^2)(v_{0n}^2 - p_{0q}^2)}, \quad (23)$$

$$Z_n = \frac{Z_1^2(v_{0n}/r)}{[r^2Z_1^2(v_{0n}) - Z_1^2(v_{0n}/r)]} \cdot \quad (24)$$

The unknown k_0 occurs within the β_m and Γ_n in the form given by (5). Let $\Delta(k_0)$ represent the value of the above determinant (20) for any particular value of the unknown k_0. Then

$$\Delta(k_0) = 0 \quad (25)$$

will give the eigenvalues, k_{0n}, of the cavity. In any numerical work the order of the determinant that will give a desired accuracy must be determined.

Error Analysis

There are two major sources of error in the infinite series expansion approach. The first is incurred in terminating the infinite series, h_{qm}, that lead to the evaluation of the expansion coefficients. The second source of error results from limiting the order of the secular determinant (20). A further consideration is the number of significant figures that must be carried in the calculations for the eigenvalues to have a desired accuracy.

We will first consider the infinite series h_{qm}. To investigate the error introduced through terminating these infinite series, the functions (21)–(24) (particularly their asymptotic behavior for large n) must be considered in some detail. Next we will consider the effect on the eigenvalues that results from considering a finite determinant in (25). Finally it will be shown how the desired accuracy of the eigenvalues determines the number of significant figures that must be carried in the calculations.

To observe the convergence of the h_{qm} we need the asymptotic values of the z_n. To obtain the asymptotic form requires the following information.

The zeros of $J_0(p)$, as p becomes sufficiently large, approach the zeros of

$$\cos(p - \pi/4), \quad (26)$$

and the zeros of

$$J_0(v)N_0(v/r) - J_0(v/r)N_0(v) \quad (27)$$

approach the zeros of

$$\tan(v/r) - \tan(v) \quad (28)$$

for v sufficiently large.

This gives

$$p_{0m} = (2m + 1)\frac{\pi}{2} + \frac{\pi}{4} \quad (29)$$

and

$$v_{0n} = nr\pi/(r - 1), \quad (30)$$

respectively, for sufficiently large m and n.

The function z_n, given in (24), then becomes

$$Z_n = \left\{\left[\frac{rZ_1(v_{0n})}{Z_1(v_{0n}/r)}\right]^2 - 1\right\}^{-1} \cong (r - 1)^{-1} \quad (31)$$

for n sufficiently large. Table I compares the exact values and the limiting value of z_n as n increases for the particular case $r = 4$.

The behavior of the functions h_{mq} will now be considered. Let a value of $n = n_0$ be selected sufficiently large so that the large argument approximation for the Bessel functions may be used. The following result is then obtained

$$h_{mq} = \sum_{n=1}^{n_0 - 1} \Gamma_n \tan(\Gamma_n L)\cdot z_n \cdot h_{nmq} + \sum_{n=n_0}^{\infty} \frac{\Gamma_n \tan(\Gamma_n L)\cdot J_0(p_{0m}/r)J_0(p_{0q}/r)}{(r - 1)(v_{0n}^2 - p_{0m}^2)(v_{0n}^2 - p_{0q}^2)} \cdot \quad (32)$$

Next a value of $n = n_1$ may be chosen such that $n_1 > n_0$ and sufficiently large to ensure

$$v_{0n}^2 \gg p_{0m}^2, p_{0q}^2 \quad (33)$$

and

$$(v_{0n}/b)^2 \gg k_0^2. \quad (34)$$

Then,

$$\Gamma_n \tan(\Gamma_n L) \cong rn\pi/(r - 1)b. \quad (35)$$

Therefore, the final term in (32), defined as the error term E_{mq}, is

$$E_{mq} = \sum_{n=n_0}^{\infty} \frac{\Gamma_n \tan(\Gamma_n L)\cdot J_0(p_{0m}/r)J_0(p_{0q}/r)}{(r - 1)(v_{0n}^2 - p_{0m}^2)(v_{0n}^2 - p_{0q}^2)}, \quad (36)$$

TABLE I

n	$z_n(r=4)$
1	0.36273077
2	0.34271069
3	0.33846524
4	0.33559658
5	0.33507737
6	0.33472793
7	0.33349480
8	0.33404574
9	0.33388359
10	0.33683268
11	0.33433147
12	0.33574898
13	0.33383231
14	0.33031662
15	0.33262065
16	0.33315857
17	0.33339393
18	0.33366988
19	0.33336504
20	0.33334554
$\lim_{n\to\infty}$	$z_n=0.33333333$

TABLE II

m, q	A $h_{mq}(n=20)$	B $E_{mq}(n=20)$	$A+B$	C $h_{mq}(n=40)$	D $E_{mq}(n=40)$	$C+D$
1, 10	0.001069	−0.00004853	0.001021	0.001033	−0.00001245	0.001021
8, 5	0.000710	0.00001531	0.000725	0.000721	0.00000394	0.000725
6, 5	0.002994	−0.00002771	0.0029663	0.002974	−0.00000776	0.0029662

and leads to the result

$$E_{mq} = J_0(p_{0m}/r)J_0(p_{0q}/r)\frac{(r-1)^2}{b(\pi r)^3}\sum_{n=n_1}^{\infty}\frac{1}{n^3}; \tag{37}$$

hence,

$$h_{mq} = \sum_{n=1}^{n_1-1} z_n h_{nmq}\Gamma_n \tan(\Gamma_n L) + \frac{(r-1)^2}{6(r\pi)^3}J_0(p_{0m}/r)J_0(p_{0q}/r)\sum_{n=n_1}^{\infty}\left(\frac{1}{n}\right)^3. \tag{38}$$

An estimate can now be made of the error incurred in terminating the series h_{mq} after a finite number of terms. This error may then be reduced through the use of the error term E_{mq}.

Table II shows some of the results that were obtained in the course of the calculations. Eq. (21) was used to compute the sum of the first twenty and forty terms in the series h_{mq}. The error in the value of the h_{mq} that occurs when we terminate the series after twenty and forty terms was then computed using (37).

If (37) gives a good prediction of the error made, then the E_{mq} added to the h_{mq} should give identical results for the two cases. Table II shows that when conditions (33) and (34) are satisfied, excellent agreement is obtained. In the particular case illustrated an accuracy of six decimal places could be achieved by summing the exact series to twenty terms and correcting this through the error term E_{mq} given by (37).

The next source of error is introduced in the solution of the secular determinant (20) for the eigenvalues k_{0n}. The eigenvalues k_{0n} that satisfy (25) are those values of k_0 that give the zero crossings of the secular determinant. To determine the k_{0n} we are not interested in the absolute value of the determinant $\Delta(k_0)$ but only in the behavior of $\Delta(k_0)$ in the neighborhood of the k_{0n}. We notice that as $L\to 0$, the $h_{mq}\to 0$, and in the limit $L=0$ we retain only the diagonal terms in the determinant, where the successive eigenvalues are obtained from the successive terms in the diagonal. The eigenvalues of the perturbed cavity are thus associated with the corresponding diagonal terms and are perturbed from their value for $L=0$ by the nonzero off-diagonal terms. It may be noted here that only the β_q and Γ_n in the determinant are functions of k_0; *i.e.*,

$$\beta_q = [k_0^2 - (p_{0q}/b)^2]^{1/2}$$

and

$$\Gamma_n = [k_0^2 - (v_{0n}/b)^2]^{1/2}.$$

When the lowest values of k_{0n} are of interest, the β_q and Γ_n, for sufficiently large q and n, become essentially independent of k_0 since than $k_0 \ll p_{0q}/b$ and $k_0 \ll v_{0n}/b$.

Also for q and m large, *i.e.*, p_{0q} and p_{0m} large, h_{mq} becomes

$$h_{mq} = \frac{2r}{\pi(p_{0m}p_{0q})^{1/2}} \cos\left(\frac{p_{0m}}{r} - \frac{\pi}{4}\right) \cos\left(\frac{p_{0q}}{r} - \frac{\pi}{4}\right) \cdot \sum_{n=0}^{\infty} \frac{\Gamma_n \tan(\Gamma_n L) \cdot Z_1^2(v_{0n}/r)}{[r^2 Z_1^2(v_{0n}) - Z_1^2(v_{0n}/r)](v_{0n}^2 - p_{0m}^2)(v_{0n}^2 - p_{0q}^2)},$$

where we have shown that the summation converges, and therefore as q and $m \to \infty$ so $h_{qm} \to 0$. Furthermore, under these same conditions

$$x_q \cong \beta_q \tan \beta_q(H - L) \cos^2\left(p_{0q} - \frac{\pi}{4} - \frac{\pi}{2}\right) \Big/ 2p_{0q}^2\pi$$

$$= \beta_q \tan \beta_q(H - L) \cos^2\left[(2q + 1)\frac{\pi}{2} - \pi\right] \Big/ 2\pi p_{0q}^2$$

Therefore $x_q \to 0$ for sufficiently large q.

Thus, not only are the coefficients h_{qm} and x_q insensitive to changes in k_0 for q and m sufficiently large in the range where the lowest eigenvalues lie but, also, these coefficients tend to zero as their order increases.

Since we are interested only in the behavior of $\Delta(k_0)$ as a function of k_0, we need to examine only that part of the determinant that is sensitive to changes in the value of k_0 being considered. For the lowest eigenvalues of k_0 we will therefore approximate the infinite determinant by a finite one since the further terms in the determinant can only serve to change the scale of the results and not their functional form. The size of the determinant is determined by inspecting the sensitivity of the coefficients to changes in k_0 as they are being calculated. The column of coefficients, where the change is less than one unit in the last significant figure, forms the final column and row of the determinant that need be considered; *i.e.*, if δk_0 is the accuracy desired in k_0 then the largest m and q are chosen such that

$$h_{mq}(k_0 \pm \delta k_0) - h_{mq}(k_0) < \epsilon$$

and

$$x_q(k_0 \pm \delta k_0) - x_q(k_0) < \epsilon, \tag{39}$$

where ϵ denotes a change of less than one in the last significant figure carried in the calculations. Whichever, m or q, is the larger in order that both conditions (39) be satisfied, determines the final row and column of coefficients that will terminate the determinant to be used in the calculations.

Let us next consider the effect of any residual error in $\Delta(k_0)$ on the values of k_{0n} so obtained. Assume errors δ_1 and δ_2 in the values of $\Delta(k_0^{(1)})$ and $\Delta(k_0^{(2)})$ (where $\Delta(k_0^{(1)})$ and $\Delta(k_0^{(2)})$ are the correct values of the determinant for $k_0 = k_0^{(1)}$ and $k_0 = k_0^{(2)}$, $k_0^{(1)} < k_0$ and $k_0^{(2)} > k_0$). The $k_0^{(1)}$ and $k_0^{(2)}$ are selected such that the root k_{0n} of the determinant $\Delta(k_0)$ is straddled by the two values of k_0. If δk_{0n} is the error in the value of k_{0n} due to the presence of δ_1 and δ_2 and linear interpolation is presumed acceptable as far as indication of the error is concerned, then for $|\delta| \ll |\Delta|$ we obtain

$$\delta(k_{0n}) \cong \frac{\delta_1 \Delta(k_0^{(2)}) - \delta_2 \Delta(k_0^{(1)})}{[\Delta(k_0^{(1)} - \Delta(k_0^{(2)})]^2} \cdot (k_0^{(2)} - k_0^{(1)}). \tag{40}$$

Therefore, if the δ_n are at least one order of magnitude smaller than $\Delta(k_0^{(n)})$ and $\Delta(k_0^{(n)}) \approx \Delta(k_0^{(n+1)})$, the error in k_{0n} is at least one order and generally two orders of magnitude less than $(k_0^{(2)} - k_0^{(1)})$.

Finally, the number of significant figures that must be carried in the calculations is determined, of course, by the accuracy desired in the results. Once we have determined the number of significant figures required in the k_{0n}, we then inspect the terms containing k_0 and assure that a sufficient number of significant figures are carried in the calculations, so that the terms around the diagonal term in the secular determinant that is associated with the eigenvalue sought are sensitive to changes in the last significant figure of k_0. Directly from this we can proceed with the tests (39) which tell us how large a determinant will have to be considered in our calculations.

Results

Calculations were performed on a circular cylindrical cavity. The cavity parameters assumed were: inside length $2H = 0.146$ m, inside diameter $2b = 0.0491$ m, and $r = 4$. The ratio L/H was varied over the range $0 \to 0.2$. Four-by-four and ten-by-ten determinant approximations were used to calculate the resonant frequencies of the two lowest TM (even) modes, *i.e.*, the TM_{010} and TM_{012} modes, as the ratio L/H was varied from 0 to 0.2. These results are shown in Figs. 2 and 3 together with the curves obtained from Slater's perturbation theory and the experimental data. The relatively simple experimental equipment used to obtain the points shown provided an accuracy no greater than three significant figures. A Datatron-204 digital computer was used throughout these calculations. The computing time required for evaluating $\Delta(k_{0n})$ for a particular value of L/H and k_0 was somewhat less than three minutes for the ten-by-ten approximation.

From the results shown in Figs. 2 and 3, k_s was found to be 3.63 and 3.67 for the two lowest modes. Since k_s is only a function of the geometry of the perturbing object and the mode type considered, it is to be expected that the values of k_s change little when the order of the mode is changed, providing the field structure in the neighborhood of the perturbing object does not change radically. The calculation of k_s is reasonably rapid since, for small L/H ratios, a four-by-four determinant approximation is usually found to suffice.

The poor agreement of the ten-by-ten approximation in Fig. 3 is caused, to some extent, by the fact that only five significant figures were used in calculating these results. However, the main conclusion is that a ten-by-ten approximation is not sufficient to satisfy conditions (39) for L/H greater than 0.1 for the second mode. Extrapolating the curve from small values of L/H for the

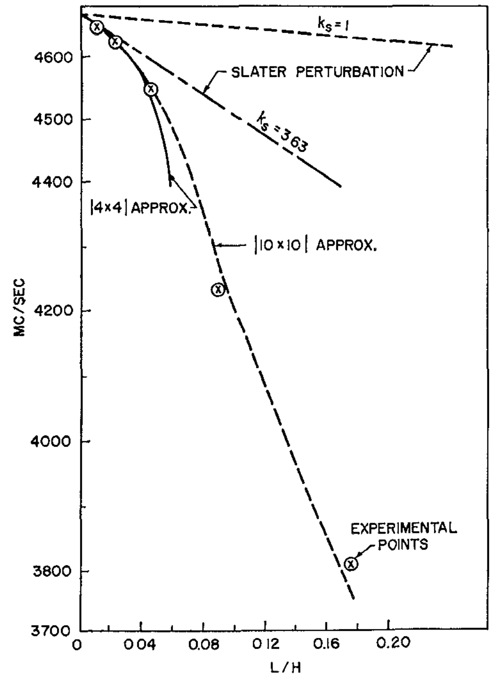

Fig. 2—Resonant frequency vs L/H, 1st mode TM_{010}.

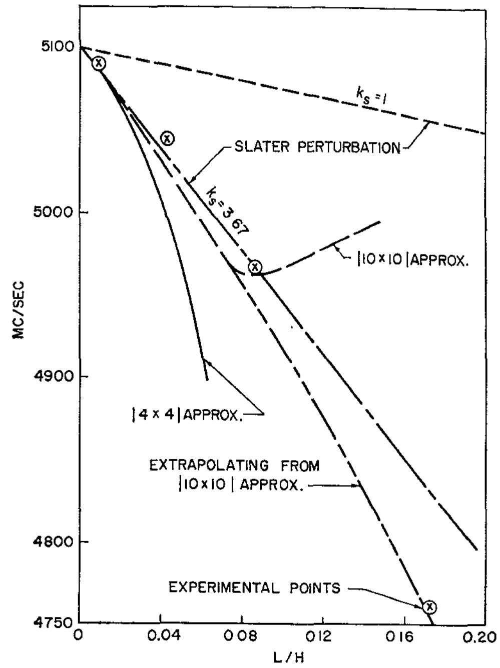

Fig. 3—Resonant frequency vs L/H, 2nd mode TM_{012}.

latter case gives reasonable agreement with the experimental results up to the limit of the data obtained.

Conclusions

It may be concluded that this method, as used by previous authors to determine discontinuity impedances may be extended successfully to determine the eigenvalues of centrally loaded circular cylindrical cavities. The necessary accuracy can be achieved rapidly on high-speed digital computers without a need to generate special functions to increase the rate of convergence. Furthermore it has been shown that this method of calculation allows a check of the number of terms to be used in the series expansions, the order of the determinant, as well as the number of significant figures that must be carried in the calculations for a desired degree of accuracy.

Table of Symbols

a_m, b_n, c_n	constants
A_m, B_n	constants
E_{mq}	error term in h_{mq}
H, L, b, a	dimension of the cavity
$r = b/a$	
K_t	a constant defined by a ratio of integrals
$k_n, k_m, \beta_m, \Gamma_n$	constants of separation
$k_0 = \omega\sqrt{\mu_0\epsilon_0}$	
$k_0^{(1)}, k_0^{(2)}$	particular values of k_0
k_{0n}	the set of eigenvalues of the system under consideration
k_s	proportionality constant
m, n, q	integers
n_0, n_1	particular values of n
p_{0m}	zeros of $J_0(p)$
R_{Aq}, R_{Bn}	sets of constants
v_{0n}	zeros of $Z_0(v) = Z_0(v/k)$
V_t	total cavity volume
$x_q, z_n, h_{mq}, h_{nmq}$	functions
$Z_n(x)$	linear combination of Bessel function of the first and second kind
$\Delta(k_0)$	indicial determinant
Δt	volume of perturbing body
ω	radian frequency
ω_0	resonant radian frequency of undisturbed system
δ_1, δ_2	error in $\Delta(k_0)$
$\gg$	"very much greater than."

Acknowledgment

The author is indebted to Profs. W. H. Hayt, F. V. Schultz and I. Marx of Purdue University for their help and encouragement.

Bibliography

[1] J. R. Whinnery and H. W. Jamieson, "Equivalent circuits for discontinuities in transmission lines," Proc. IRE, vol. 32, pp. 98–115; February, 1944.

[2] J. R. Whinnery, H. W. Jamieson, and T. E. Robbins, "Coaxial line discontinuities," Proc. IRE, vol. 32, pp. 695–710; November, 1944.

[3] J. R. Whinnery and D. C. Stinson, "Radial line discontinuities," Proc. IRE, vol. 43, pp. 46–51; January, 1955.

[4] W. C. Hahn, "A new method for calculation of cavity resonators" *J. Appl. Phys.*, vol. 12, p 62; January, 1941.

[5] E. Jahnke and F. Emde "Tables of Functions," Dover Publ., Inc., New York, N. Y., p. 167; 1945.

[6] "National Bureau of Standards Mathematical Tables, Tables of Function and Zeros of Functions," Natl. Bur. Standards, Washington, D. C.; November, 1954.

[7] J. C. Slater, "Microwave Electronics," D. Van Nostrand Co., Inc., New York, N. Y., pp. 81–83; 1950.